Forecasting Timber Growth by the Point Center Extension Modification of the Bitterlich System*

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INTRODUCTION

PURPOSE

One of the most important items of basic data for the preparation of a forest management plan or forest appraisal is the growth estimate. The methods that are in current use include the movement radio method (6), the abreviated method (9), growth percent methods and the recurrent inventory systems (6). The movement ratio method is particularly laborious to apply. It requires the construction of graphs from the raw data in order to find the correct anticipated growth in diameter, taking a great amount of time during the application of the movement ratio formula. This method assumes that the trees are evenly distributed by size throughout each diameter class and then applies an average growth rate to all trees in the class.

The abreviated method (9) is a variation of the stand table projection methods. It will be discussed later in the study, since it was used as the standard method to be compared with the point-center extensión method.

The growth percent methods are an expression of the tree volume growth during a given growth period as a percentage of volume at the beginning of the period. Since volume growth is a function of change in diameter, height and form, mathematical treatment, in terms of these factors, is possible. There have been many formulas developed for this purpose and all of these are conservative and for current growth only. They ignore the quality and consequently price increment which is very important. They express past growth, rather than determining what future growth will be (6). The main difficulty with these methods is that volume growth is not a quantity but a simple relationship with the shifty properties of percentages in general. As the tree grows larger the percentage growth rate tends to decline, since growth is measured against a constantly increasing total volume base.

Recurrent inventory systems (6) are procedures for obtaining and currently maintaining information on current growth, yield and mortality as obtained from the stand itself (6). The main advantage of these methods is that the sampling error is eliminated by the establishment of permanent plots. Also, the accuracy of the information obtained is greater than in most of the other methods. The main disadvantages are the high cost and technical problems involved in getting and obtaining an adequate sample, high field and office analysis costs, the necessity of maintaining high consistency and uniformity in the measurements and the tendency of the method to be inelastic in meeting changing conditions in management and utilization practices (6).

All the methods mentioned above have satisfactory accuracy but they have deficiencies which limit their use in many forest management operations. One of their main handicaps is the time involved in the calculations and in the construction and application of graphs from the data. In mathematical methods of growth prediction the time involved in the calculation of growth is very large, increasing the cost of the work. This increased cost of the growth estimate makes the work prohibitive for many reconnaissance-type timber stand surveys.

One recently developed method that is rapidly being adopted in forest inventory work is the Bitterlich method. Also called plotless inventory, angle-count method or variable plot radius cruising (V.P.R.), it has been used to collect data for many different purposes: forest management, silvicultural condition analyses, and ecological investigation, to mention a few. Providing for the rapid computation of basal area in square feet per acre and volume per acre, the system is well adapted to field reconnaissance procedures.

In 1963, Fender and Brock (7) adapted the Bitterlich system to growth estimation. This innovation which they named the point-center extension method, shows great promise for the rapid field-estimation of current economic growth. This method is based on establishing the average diameter growth rate of the trees in the vicinity of the sampling point and then using this to project the center of that plot in order to predict which trees will grow enough to be included in the future inventory.

The purpose of this tudy was to test the accuracy of the point-center extension method, and, if necessary, to develop modifications that would increase its accuracy and still maintain its economical benefits.

LITERATURE REVIEW

In 1948, a new theory applied to the computation of basal area per acre was introduced to forest science by Dr. W. Bitterlich (1). This new method may revolutionize the computation of basal area, volume and growth in forest stand inventory.

As foresters applied the method, they developed many modifications, making it more versatile and more accurate. One of the most

recent modifications is concerned with the forecasting of timber growth.

When Bitterlich explained his theory, he based it on the tree-diameter/spacing ratio by which he counted all the trees in a circle that exceeded the ratio (Winkelzahl) of 1.41. The purpose is to distingui.h between readings greater than and less than 1.41. Basal area in square meters per hectare is then obtained directly for each species by halving the numbers so obtained. This procedure is based on the observation that in such a circle the sum of all measurable ratios exceeding some assumed limiting value, is proportional to the basal area of the stand. The smaller this limiting value, the greater the number of stems to be counted, the larger the circle to be scanned, and the more accurate the results.

This original method was useful in terrain with a reasonably-uniform slope. For sloping terrain, corrections have to be made by dividing the results by the cosine of the angle of the slope.

Later, in 1949, Bitterlich (2) made one modification to his original angle-count method, in which he eliminated the term "diameter" (d) from the fundamental equation:

"When in a stand of trees uniform in d but arranged in any manner, we select any limiting Winkelzahl value, Wg, and take sightings all around on trees satisfying this value, we describe a circle of radius $\frac{100}{\mathbf{Wg}}$ .d, and all trees for which $d > $ Wg will lie within the circle. Now the basal area of the trees within the circle will be related to its own area

$$\left(\frac{\mathbf{d}}{\mathbf{2}}\right)^2 \bigg/ \left(\frac{\mathbf{100}}{\mathbf{wg}}\text{-d}\right)^2$$

or more simply as $\dfrac{1}{\mathbf{10.000}}\left(\dfrac{\mathbf{2}}{\mathbf{wg}}\right)^2$, where d is eliminated.

If Wg $= 2$, each tree counted within the circle will have a basal area $\dfrac{1}{\mathbf{10.000}}$ that of the circle itself, i.e. will correspond to a b.a. at 1 sq. m./ha. On level ground, ba/ha $= \left(\dfrac{\mathbf{wg}}{\mathbf{2}}\right)^2$ x number of trees in circle.

As d has been eliminated from this fundamental equation, the latter is valid not only for the case of a stand of uniform tree-diameter but also for any mixture of d.b.h. values ocurring in the forest. It must always be remembered in practice that the radius of the sample area is proportional to the d.b.h. values.

The coefficient $\left(\dfrac{wg}{2}\right)^2$, of Zahlfaktor ("Count-factor"),

enables the appropriate multiplier to be found for calculating b.a./h.a. from number of trees, for any value of Wg (2)".

The accuracy of this method is regarded as being as good as those of stem-by-stem calipering, provided that the instrument used is not faulty and that no too many "borderline" trees are found.

In 1952, L. R. Grosenbaugh (10), introduced the theory of Bitterlich's method to American foresters. At this stage, the method had been used to determine basal area in square feet per acre, volume per acre, average volume per tree, etc. Grosenbaugh introduced the concept of plot radius factor (P.R.F.) which when multiplied by the tree diameter in inches gives the distance the tree can be from the point center, in feet, and still be a count tree. He also defined the basal area factor (B.A.F.) A B.A.F. of 10, por example, means that each tree that subtends an angle larger than 104.18 minutes will represent 10 square feet of basal area per acre in the vicinity of the sample point. A change in the plot radius factor, as a result of changing the critical angle of 104.18 minutes will change the B.A.F. However, this factor can be readily computed for various critical angles and then applied to obtain the basal area per acre contribution of each count tree.

When Bitterlich (1) developed his theory, and in order to count the Winkeltzahlen (ratios) exceeding 1.41, he used a 1-meter stick with a clearly marked measuring line of 1.41 cm. long, on the end of the stick to define the critical angle. This stick called "Bitterlich's optical fork", was converted by Grosenbaugh to a stick 33″ long with a one-inch measuring device mounted on the end. Such an instrument has a critical angle of 104.18 minutes, and defines a circle with a radius exactly 33 times the diameter of any tree under consideration. The arrangement is such that where the observed sample tree exactly subtends the angle formed by the line of sight around the edges of the one-inch target, mounted at right angles to the line of sight exactly 33 inches from the eye, the distance from the observer's eye to the tree is exactly 33 times the tree's diameter. If the tree subtends a larger angle than that formed by the fork, then it is definitely closer to the sample point than 33d, and vice-versa.

A more convenient instrument for this determination is the wedge prism, which establishes the proper angle by displacement of the tree's image (4). Each of these prisms is measured in prism-diopters which is a "right-angled deflection of one unit in one-hundred units distance" (4). For example a 3.03 diopters prism will give the basal area in square feet per acre.

From this stage on, the Bitterlich method itself has undergone very few modifications although more precise instruments have been developed. Bitterlich's spiegelrelaskop (3), is the most advanced and most precise of these developments. In addition to providing several different critical angles for basal area per acre determination, it has scales for performing the following:

a) Calculating tree heigh
b) Measuring tree diameter at any height
c) Performing direct tree form-height measurements
d) Reading topographic corrections
e) Measuring vertical angles in either percent or degrees.

Also, it may be used as rangefinder for measuring 66-foot, 99-foot and other horizontal distances (5).

DESCRIPTION OF THE STUDY AREA

The data used in this work were collected in the Austin Cary Memorial Forest, which is located about 11 miles northeast of Gainesville, Florida on State Road 24.

The forest consists of some 2000 acres and is primarily a Florida pine flatwoods type, consisting of longleaf pine (*Pinus palustris,* Mill.), slash pine (*Pinus elliottii* var. *elliottii,* Engelm), *loblolly pine* (*Pinus taeda* L.) and some hardwoods. The study plots were located in areas were slash pine was the prevalent species.

PROCEDURE

FIELD WORK

Fifty circular plots of 1/4-acre area each were selected within the Austin Cary Memorial Forest. Since the purpose was to compare the Bitterlich Method with some standard method, one regular 1/4-acre plot inventory and one V.P.R. (Bitterlich Method) count were made from each plot center. After each V.P.R. count was performed, the point center extension count (described below) was made and recorded.

An attempt was made to place each sample plot in an area having a uniform distribution of trees. Also, an attempt was made to sample a wide range of density classes. This enabled the testing of the method under different density classes.

The following equipment was used to collect the field data: diameter tape, caliper, Pressler's increment borer, bark gauge, perforated tape, special 6-foot range pole and the spiegelrelascope (Figure 1). The use of some of the equipment will be explained subsequently.

Selection and Measurement of Sample Trees

In each plot, the five pine trees located nearest the plot center were selected as sample trees. The following measurements were taken on these trees:

1. Diameter at breast high to the nearest tenth of an inch
2. Total height to the nearest foot
3. Height to the crown center of gravity (Fogelberg's Height) to the nearest foot (8)

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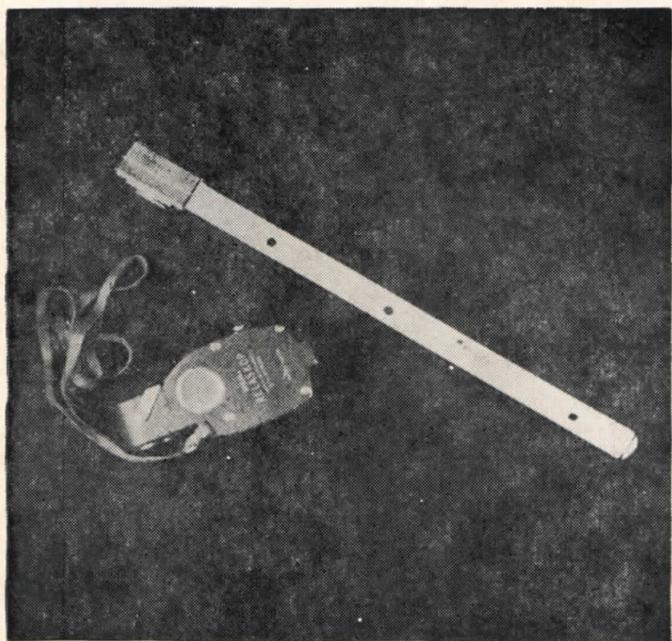
Figure 1.—Part of the Equipment Used in the Point Center Extension. The equipment shown is the spiegelrelascope and the perforated tape.

4. Radial wood growth in the past 5 years, to the nearest tenth of an inch
5. Age
6. Double bark thickness to the nearest tenth of an inch.

All the sample trees were slash pine.

Regular 1/4 Acre Plot Inventory

The trees were grouped into one-inch diameter classes and all the trees from three inches and larger were measured. The d.b.h. of each tree was measured with a tree caliper.

Point Sampling (V.P.R.) and Point Center Extension

The V.P.R. and point center extension counts were made using the same center as that of the selected 1/4 acre plot. A staff with a screw in the top was used to mount the spiegelrelascope in position on the point center, and later to hold the perforated tape used in the point center extension (Figure 2).

Using the basal area factor 10 scale of the spiegelrelascope count trees were recorded by one-inch diameter classes.

After the first V.P.R. count was completed, the point center was extended and another count made (Figure 3).

The point center extension method is based upon establishing the average diameter growth rate of the trees in the vicinity of the sampling point and then using this to predict which trees will grow enough to be included in the future inventory (7). The distance that a tree can be from the center of the plot, before the prism excludes it, depends upon its diameter. This distance, which is constant for any prism with a given basal area factor of 10, has a plot radius factor of 2.75, which means that a tree can be 2.75 feet from the center of the plot for every inch in diameter and still be counted. Therefore, if a tree grows one inch in diameter in 5 years, it can be 2.75 feet farther from the center of the plot five years hence and still be included in the future tally (7). Then, by moving the relascope 2.75 feet closer to each tree for every inch in anticipated diameter growth, we measure the basal area as it will appear five years hence. This does not measure the growth of the count trees of the first measurement, since each tree represents the same basal area in both the actual and future inventories; it simply adds to the count of the second measurement those trees that are expected to grow enough to be counted in the new inventory. Also, mortality will be automatically taken into consideration by excluding the trees that are dead at the time of the first count (7).

To accomplish the point center extension a perforated steel tape was used. The holes were spaced at 0.275 feet to account for every tenth of inch in diameter growth (Figure 1).

It must be remembered that the constant of 2.75 feet (plot radius factor) corresponds only to a prism with a basal area factor of 10.

Table 1 contains the corresponding extensions to be used for the particular diameter growth in inches during the past five years. Once the first count was completed and the average diameter growth was known, the proper extension was read from Table 1 and the particular hole in the steel tape was fixed to the top of the staff. Then the spiegel-relascope was attached to the end of the tape and a new circular sweep made to determine the new count. (See Figures 2 and 3). The difference between the second and first count, multiplied by the B.A.F. used, represents the anticipated basal area growth for the future period.

OFFICE WORK

Plots were first grouped into density classes with a range of 20 square feet, with the exception of the first and last classes, which extended from 21-60 and 121-160 square feet per acre, respectively. A wider range was necessary in these two classes due to the limited numberber of plots that actually were sampled in these extremes of density.

The purpose of this grouping was to analyze the influence of stand density on the accuracy of the method.

Sample trees provided the data necessary to compute the volume in cords, and the different rates of growth in basal area for the different plots.

The following calculations were performed for each density class:
a. Average diameter breast high in inches
b. Average total height in feet
c. Average height to the crown center of gravity (Fogelberg's height in feet)
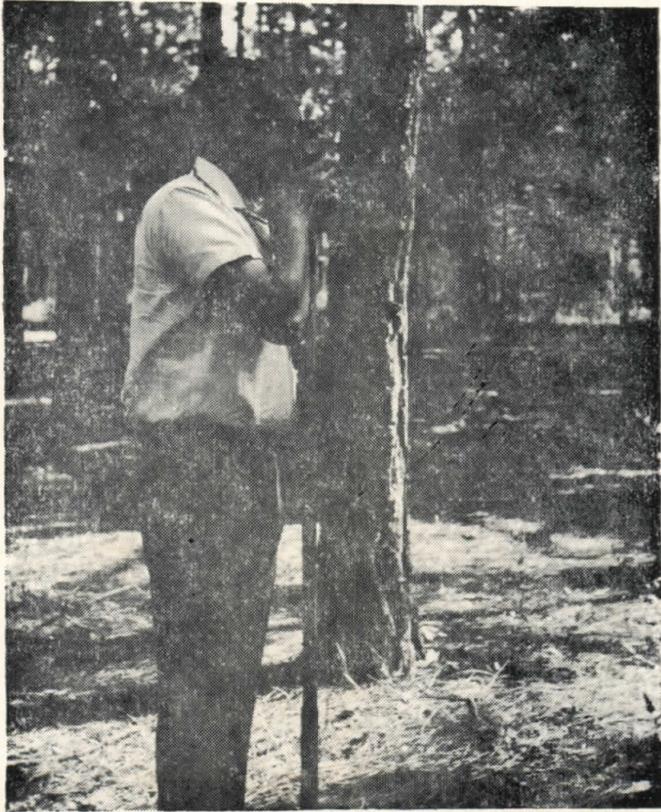
Figure 2.—First Count. Note staff to put the spiegelrelascope
at eye level.

Figure 3.—Second Count or Point Center Extension. Note perforated tape attached to the staff and hole in horizontal position.

TABLE 1

Point Center Extensions

Mean Diameter Growth in Five Years (In.)	Point at Which Holes Are Punched in Tape
0.2	0 ft. 6.6 in.
0.3	0 ft. 9.9 in.
0.4	1 ft. 1.2 in.
0.5	1 ft. 4.5 in.
0.6	1 ft. 7.8 in.
0.7	1 ft. 11.1 in.
0.8	2 ft. 2.4 in.
0.9	2 ft. 5.7 in.
1.0	2 ft. 9.0 in.
1.1	3 ft. 0.3 in.
1.2	3 ft. 3.6 in.
1.3	3 ft. 6.9 in.
1.4	3 ft. 10.2 in.
1.5	4 ft. 1.5 in.
1.6	4 ft. 4.8 in.
1.7	4 ft. 8.1 in.
1.8	4 ft. 11.4 in.
1.9	5 ft. 2.7 in.
2.0	5 ft. 6.0 in.
2.1	5 ft. 9.3 in.
2.2	5 ft. 0.6 in.

d. Average double bark thickness in inches
e. Average form point (8)
f. Percent of bark
g. Inside bark form class
g. Inside bark form class
h. Outside bark form class

Volume in cords per acre was determined using a volume formula developed by Swinford (13) from volume curves derived by Fogelberg (8).

For the computation of average d.b.h. in inches and average total height in feet, the usual procedure for the computation of a weighted average was followed.

The height above the ground of the center of gravity of the tree crown was measured on each sample tree. This height, expressed as a percentage of the total height of the tree, is the form point, which expresses the taper of the tree and is related to one of the variables in the volume equation (8).

The percent of bark is a ratio of the average double bark thickness expressed as a percentage of the average d.m.h.

The inside bark form class can be read from a special table provided by Fogelberg (8). This is more properly known as absolute form class, and is the percentage relationship between the diameter inside bark at half the distance between the breast high point and the tip of the tree and d.b.h. inside bark.

The outside bark form class can be estimated from inside bark form class and d.b.h. bark thickness measurements. The procedure is as follows: The double bark thickness expressed as a percent of the d.b.h., which is already known, will give a factor which applied to the inside bark form class will give the outside bark form class.

The actual steps to compute volume in cords/acre and the table with the sample tree data related with volume can be found in the Appendix.

Computation of Growth by the V.P.R. Method

The point center extension method was developed specifically for growth forecasting in the field. The rather simple procedure is as follows:

a. Compute the basal area per acre of the existing stand (first count) by multiplying the number of count trees, regardless of diameter, by the basal area factor of the prism or other device being used (in this case, 10).
b. From the average diameter growth in the past five years read in Table 1 the corresponding extension to be applied to the center of the plot.
c. Perform the extension and compute the basal area per acre of the new count following the same procedure explained in *a*.
d. Subtract the basal area per acre of the first count from the

basal area per acre of the second count. The difference is the growth in basal area in square feet per acre, expected in the next five years (7).

Example:

First count: 10 trees

10 x B.A.F. = 10 x 10 = 100 square feet of basal area per acre

Second count: 12 trees

12 x B.A.F. = 12 x 10 = 120 square feet of basal area per acre

Expected growth in the next five years

120 — 100 = 20 square feet of basal area per acre

For comparison with the modification of the Bitterlich method, the abreviated method for current growth computation was used as the standard method (9).

Computation of Growth by the Abreviated Method

The abreviated method (9) is a variation of the stand table projection method. Its basis is the number of years required for the average tree of the stand to grow one inch in d.b.h. (outside bark measurements).

The procedure used for this is described in the following paragraph.

First, the data were averaged and computations completed to obtain the basic information shown below:

a. D.B.H. class, in one-inch classes
b. Average present d.b.h. outside bark in inches
c. Average double bark thickness in inches
d. Average present d.b.h. inside bark in inches
e. Average diameter growth in inches
f. Average d.b.h. inside bark, five years ago, in inches
g. Average d.b.h. outside bark, five years ago, in inches
h. Average d.b.h. growth, outside bark, during the past five years
i. Number of years required to grow one inch in diameter, outside bark
j. Weighted number of years to grow one inch in diameter, outside bark.

The computation of the average d.b.h. inside bark, in inches, was achieved by subtracting the weighted average double bark thickness, in inches, from the average d.b.h. outside bark, in inches.

The ratio of outside bark d.b.h. to inside bark d.b.h. was determined by the formula:

$$\frac{\text{Sum of present d.b.h., outside bark}}{\text{Sum of present d.b.h., inside bark}} = \frac{\text{OB}}{\text{IB}} \text{ratio}$$

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The average d.b.h. inside bark, five years ago, was found by subtracting the average diameter growth from the average present d.b.h. inside bark.

To find the average d.b.h., outside bark, five years ago, the average d.b.h. inside bark was multiplied by the $\dfrac{OB}{IB}$ ratio, previously referred to. This ratio also was used to convert the average diameter growth inside bark to average diameter growth, outside bark for the five-year period.

Having an estimate of the average diameter growth outside bark during the past five years, the number of years required to grow one inch in diameter, outside bark was calculated as follows:

$$\frac{5}{\text{Average d.b.h. growth outside bark, past five years}}$$

This was done for each separate d.b.h. class.

The last term of the list is the number of years required to grow one inch in diameter, outside bark, times the frequency of the diameter class, summed up through its whole range.

Finally, the overall average number of years required to grow one inch in diameter, outside bark, was found by dividing the sum of the weighted average number of years to grow one inch in diameter, outside bark, by the total number of sample trees.

The computations of the number of years required to grow one inch in diameter, outside bark for all the density classes can be found in the Appendix, Table 6.

Once the number of years required to grow one inch in diameter was known, the actual growth in basal area in square feet per acre was computed for every plot within the particular density class. In addition to this, a stand table was prepared for every plot by projecting the number of trees in every diameter class of the 1/4-acre plot to a one-acre stand. The basal area per acre was calculated from this present stand table. After this was done the stand table was projected one inch; i.e. the four-inch class was projected to the five-inch class in the future stand table, etc. The basal area in square feet per acre for the projected stand table was then computed. The difference between the two total basal areas represented the growth in whatever number of years it took for that particular density class to grow one inch in diameter outside bark.

Table 2 is an example of the stand table projection. It represents Plot 1 of the 81-100 square feet of basal area class which took 6.31 years to grow one inch in diameter outside bark. The computation of the five-year growth is performed by means of a simple proportion.

TABLE 2

Stand Table Projection for Computation of Growth in Basal Area in Square Feet Per Acre Plot Nº 1 — Density Class 81-100 Sq. Ft.[a]

Diameter Class (inches)	Present Stand Table (Trees per Acre)	Present Basal Area Per Acre (Sq. Ft.)	Future Stand Table (Trees per Acre)	Future Basal Area Per Acre (Sq. Ft.)
4	28	—	—	—
5	12	1.632	28	3.808
6	20	3.920	12	2.352
7	28	7.476	20	5.340
8	16	5.584	28	9.772
9	24	10.608	16	7.072
10	32	17.440	24	13.080
11	8	5.280	32	21.120
12	4	3.140	8	6.280
13	12	11.064	4	3.688
14	8	8.552	12	12.828
15	4	4.908	8	9.816
16			4	5.584
TOTAL		79.604		100.740

Growth in sq. ft. $= 100.740 - 79.604 = 21.136$
No. of years required to grow one inch in diameter, ouside bark: 6.31 Yrs.
Five-year growth in sq. ft. of basal area: 16.75 sq. ft.

[a]From Sample Tree Growth Analysis. Appendix, Table 6.

Computation of Basal Area in Square Feet
Per Acre by Both Standard and Bitterlich Methods

Basal area per acre was computed by the standard method by multiplying the basal area of a given diameter class by the number of trees in that class and then, summing all the different products through the whole range. The basal areas for the diameter classe; can be computed from the function $\dfrac{\text{II d}^2}{4}$ or directly read from special tables in any book of forest management or forest mensuration.

Basal area in square feet per acre was computed by the Bitterlich's method by multiplying the sum of the count trees per plot, regardless of diameter class, by the basal area factor of the prism employed.

Computation of Number of Trees
Per Acre and Average D.B.H. in Inches

The sum of the trees in the stand table of every plot is the number of trees per acre. The basal area of the average tree in each plot was computed by dividing the total basal area per acre by the total number of trees in that plot. The diameter corresponding to this average was then read from the basal area tables, giving the average d.b.h. per plot.

Discussion of Results

The function of the growth in basal area per acre was considered under two situations:

a. When the inventory is done by routine methods and only the part corresponding to the growth prediction is determined by the point center extension method.

b. When the inventory and growth prediction is based totally on the Bitterlich (V.P.R.) and the point center extension methods. The functions for every one of these situations are respectively:

$$Y = f(X_1, X_2, X_3, X_4, X_5)$$
$$Y = f(X_1, X_5)$$

Where

Dependent Variable Y:

Growth in square feet of basal area per acre as computed by the abreviated method.

Independent Variables:

X_1 = Growth in basal area in square feet per acre as computed by the point center extension method.

X_2 = Total basal area in square feet per acre as computed by the standard method.

X_3 = Number of trees per acre

X_4 = Average d.b.h. per plot, in inches

X_5 = Total basal area in square feet per acre as computed by the Bitterlich method.

The form of these functions were specified as follows:

$$Y = b_{01} + b_{11}X_1 + b_{21}X_2 + b_{31}X_3 + b_{41}X_4 + b_{51}X_5$$
$$Y = b_{02} + b_{12}X_1 + b_{22}X_5$$

The coefficients in the equations were estimated with the step-wise multiple regression program. The program permits transformations on the variables.[1]

The estimated coefficients for the step-wise regression equations can be found in Table 3.

The signs of the coefficients seem consistent with what one would logically expect. However, in the first situation the coefficients for X_2, X_4, and X_5 are not significantly different from zero at an acceptable level. In the second situation all coefficients are significant.

In the first situation less than 1% additional variation was explained with the introduction of X_2, X_4 and X_5.

On the basis of significancy of the coefficients and the amount of variation explained, little seems to be gained by using variables X_2, X_4 and X_5 in the first estimating equation.

Using equation 2 of the first situation (see Table 3), the following standard errors were computed:
Sample standard error of Y estimated from X_n = 0.28843 sq. ft.

Standard error of prediction: 3.53649 sq. ft.

Standard error of forecast: 2.88793 sq. ft.

The standard error of forecast is the possible error in the prediction of an average value of Y for a given set of X's. Its formula is:

$$S_{\bar{y}} = S_y \sqrt{\frac{1}{n} + c_{11} x^2_1 + c_{22} x^2_2 + 2c_{12} x_1 x_2}$$

Where

S_y = Standard error of regression

n = Number of samples

x_n = Corrected sum of squares

The standard error of prediction is the possible error in the the
C_{11}, etc. = Elements of the inverse or coefficients matrix[1]
prediction of a particular. Y for a given set of X's. Its formula is:

$$S_{\bar{y}} = S_y \sqrt{1 + \frac{1}{n} + c_{11}x_1^2 + c_{22}x_2^2 + 2c_{12}x_1x_2}$$

The only difference between these two formulas is the addition
of 1 to the second formula, to account for the extra variation of the
particular value of Y.

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[1]For the computation of these elements: Snedecor p. 418 (12).

$$c_{11} = \sum x_2^2 \Big/ \left(\sum x_1^2\right)\left(\sum x_2^2\right) - \left(\sum x_1 x_2\right)^2$$

$$c_{22} = \sum x_1^2 \Big/ \left(\sum x_1^2\right)\left(\sum x_2^2\right) - \left(\sum x_1^2 x_2^2\right)^2$$

$$c_{12} = \sum x_1 x_2 \Big/ \left(\sum x_1^2\right)\left(\sum x_2^2\right) - \left(\sum x_1 x_2\right)^2$$

TABLE 3

Table of Coefficients

Equation	Y Intercept	X₁	X₂	X₃	X₄	X₅	Degrees of Freedom	F Level of Variable Entering	Sy	R	R^5
First Situation:											
1	6.81372			(0.00178) 0.03932*			48	488.3699	2.2625	0.95421	0.91052
2	8.37572	(0.03736) −0.12982*		(0.00195) 0.04317*			47	12.0718	2.0394	0.96374	0.92879
3	6.52249	(0.03744) −0.11489*	(0.02054) 0.03688	(0.00345) 0.03801*			46	3.2254	1.9928	0.96616	0.93347
4	6.95448	(0.03789) −0.11470*	(0.02712) 0.03871	(0.00602) 0.03749*	(0.55256) −0.05770		45	0.0109	2.0146	0.96617	0.93348
5	18.70018	(0.03826) −0.11374*	(0.04071) 0.05355	(0.01026) −2.86772	(5.73472) −2.86772	(0.32916) 0.16206	44	0.2424	2.0318	0.96636	0.93385
Second Situation:											
1	5.14178					(0.02445) 0.16655*	47	3.0974	5.2793	0.72311	0.52289
2	3.36972	(0.08607) 0.15148*				(0.02589) 0.14918*	48	46.3863	5.3934	0.70104	0.49146

NOTE: The number in parenthesis on top of every coefficient is the standard error of coefficient (Sb). The coefficient of determination (R^2) expressed as a percent denotes the amount of the variation «explained» by the regression. The asterisk denotes significancy at an alpha level of 0.05.

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The different values of the inverse matrix were as follows:

$c_{11} = 0.00027$
$c_{12} = 0.00003$
$c_{22} = -0.00003$

Finally, 95% confidence intervals were calculated for both prediction of an average Y and a particular Y. The formulae used were:

$$Y \pm t_{.05} \; Sy \sqrt{\frac{1}{n} + c_{11} x_1^2 + c_{22} x_2^2 + 2c_{12} x_1 x_2}$$

$$Y \pm t_{.05} \; Sy \sqrt{1 + \frac{1}{n} + c_{11} x1_2 + c_{22} x2_2 + 2c_{12} x_2}$$

and the results:

Y + 5.8163 square feet

Y + 7.1125 square feet

Using equation 2 of the second situation (see Table 3) the following standard errors were computed:

Sample standard error of Y estimated from X_n = 0.74661 sq. ft.

Standard error of forecast: 7.09010 sq. ft.
Standard error of prediction: 8.83755 sq. ft.

In this case the coefficient of determination (R^2) tells that the regression explains about 53% of the total variation in comparison with 93% explained in the first situation. The decreased amount of variation explained by this equation is probably due to the characteristic of the Bitterlich method of working only in integer numbers, in this case, both the basal areas and growth were expressed in groups of 10 square feet each. This lack of continuity in the growth curve contributes a great deal to the variation.

The values of the inverse matrix, por the second situation, were:

$c_{11} = 0.00026$
$c_{12} = -0.00003$
$c_{22} = 0.00002$

The 95% confidence intervals:

Y + 14.27946 sq. ft.

Y + 17.79883 sq. ft.

The greatest deviations between the actual growth and predicted growth occured in plots having the highest density (Plots N° 28, 37, 38, 47, from the density classes of 101-120 and 121-160 square feet of basal area).

Growth as predicted by the point center extension method was negatively correlated with basal area as computed by the V.P.R. method (Table 4). Growth as predicted by the standard method (abreviated), showed a positive correlation with the basal area as computed by the standard method, or in other words, the denser the stand the greater the amount of growth in basal area. In dense stands borderline and hidden trees decrease accuracy of the count. This factor may explain the correlation.

The average d.b.h. in inches has a very highly significant correlation with the basal area per acre as computed by the Bitterlich method; the square of the d.b.h. is highly correlated with the total basal area as computed by the standard method. Also, the greater the number of trees per acre, the smaller the average d.b.h. in inches and, the higher the basal area per acre as computed by the standard method.

Table 4 shows all the simple correlation coefficients among the different variables used in this study.

TABLE 4

Simple Correlation Coefficients

Variables	Y	X_1	X_2	X_3	X_4	X_5
First Situation						
Y	1.000000	0.431163	0.864662	0.954206	—0.711523	—0.679624
X_1		1.000000	0.389875	0.568454	—0.500905	—0.477013
X_2			1.000000	0.853370	—0.456697	—0.437949
X_3				1.000000	—0.781930	—0.747108
X_4					1.000000	0.995600
X_5						1.000000
Second Situation						
Y	1.000000	0.431163				0.701036
X_1		1.000000				0.381196
X_5						1.000000

NOTE: All correlation coefficients are significant at an alpha level of 0.05.

In the process of investigating various applications of the point center extension method for the prediction of growth in basal area per acre of a forest stand, two situations were encountered:

a. When the forest inventory was carried out under routine practices and only the growth was predicted using the point center extension.

b. When the study of the forest —as performed entirely with the V.P.R. method, using point center extension for the prediction of growth in basal area per acre.

The first situation required more elaborate computational work; it is more accurate, and more costly than the second situation. The second equation of Table 3 for the first situation includes, beside the growth predicted by the point center extension, the total number of trees per acre. This increases the accuracy in growth prediction from a correlation coefficient of 0.431163 (abreviated and point center extension) to a correlation coeficient of 0.96374. This increase represents a gain in the percentage of variation explained in the regression from 19% to 93%.

This high accuracy is somewhat reduced in the second situation mentioned. In this case, the equation for the prediction of growth includes the basal area per acre as computed by the V.P.R. method. This tends to produce errors in stands with extreme densities. Despite this reduced accuracy, the coefficient of correlation still permits a reliable growth prediction. The inclusion of the basal area as computed by the V.P.R. method increase the percentage of the variation explained from 19% to 52%.

In both situations, the application of the point center extensión is less costly than the standard method. The computation of growth in basal area per acre by the second situation results in a saving of about 60% in time, while the first situation saves about 40% over conventional growth estimation procedure.

The point center extension is very simple and easy to understand and apply in the field. It can be used in most situations of topography and densities, provided an instrument which automatically corrects for slope is employed in making the tree count. The spiegelrelascope is highly recommended for plot center extension work, particularly in broken terrain since it has a built in device that automatically corrects for slope. Besides that, it is very easy to work with, easy to carry and very accurate in measurements.

Table 5

SAMPLE TREE DATA IN PRESENT VOLUME COMPUTATION

Density Class	D. B. H. Class	Average D. B. H:	Total Height	Fogelberg's Height	Average D. B. T.	Average Form Point	Percent of Bark	Inside Bark Form Class	Outside Bark Form Class	Nº of Samples
Square Feet of Basal Area	Inches	Inches	Feet	Feet	Inches		%			Trees
21 - 60	4	4.10	35.0	25.0	0.70	71	17	69.5	66	1
	5	4.90	37.0	30.0	1.10	81	22	74.5	68	1
	6	6.43	42.3	32.3	1.13	76	18	72.0	68	3
	7	7.20	47.3	31.0	1.23	66	17	67.0	64	3
	8	8.07	49.4	34.0	1.31	69	16	68.5	65	7
	9	8.93	52.8	35.8	1.30	68	15	68.0	65	9
	10	10.00	52.4	32.9	1.63	63	16	65.5	62	7
	11	11.37	53.7	36.7	1.50	68	13	68.0	67	3
	12									
	13	13.30	60.0	45.0	1.60	75	72	72.5	71	1
61 - 80	4	4.35	36.0	30.0	1.35	83	31	75.5	64	2
	5	5.13	40.0	31.2	1.08	78	21	73.0	67	6
	6	5.89	43.9	31.6	1.07	72	18	70.0	66	7
	7	6.97	49.8	35.5	1.19	71	17	69.5	66	15
	8	8.06	52.0	36.7	1.25	71	16	69.5	66	10
	9	8.99	49.6	35.8	1.21	62	13	70.0	69	9
	10	10.04	44.4	40.4	1.40	63	14	70.5	68	8
	11	11.14	57.5	42.1	1.64	73	15	70.5	68	8
	12	12.13	61.7	43.3	1.73	70	14	69.0	67	3
	13	12.75	60.0	43.5	1.90	72	15	70.0	67	2

TABLE 5 *Continued*

Density Class	D. B. H. Class	Average D. B. H:	Total Height	Fogelberg's Height	Average D. B. T.	Average Form Point	Percent of Bark	Inside Bark Form Class	Outside Bark Form Class	Nº of Samples
Square Feet of Basal Area	Inches	Inches	Feet	Feet	Inches		%			Trees
81 - 100	4	4.15	26.5	22.2	0.70	84	16	76.0	72	4
	5	5.12	40.5	29.7	0.95	73	19	70.5	66	4
	6	6.03	46.7	36.7	1.17	79	19	73.5	68	3
	7	7.11	49.6	34.7	1.21	70	17	69.0	66	9
	8	8.06	55.6	42.3	1.27	76	16	62.0	68	11
	9	9.11	55.8	40.4	1.30	72	14	70.0	68	14
	10	10.05	60.5	45.8	1.32	76	13	72.0	71	16
	11	10.99	63.6	46.1	1.41	72	13	70.0	69	7
	12	11.93	63.7	46.7	1.47	73	12	70.5	69	6
	13	13.10	70.0	52.5	1.97	75	15	71.5	69	4
	14	14.40	65.0	40.0	2.50	62	17	65.0	62	1
	15									
	16									
	17	16.90	65.0	40.0	3.00	62	18	65.0	61	1
101 - 120	4	4.15	38.3	30.9	0.75	81	18	74.5	70	10
	5	5.14	36.7	29.0	1.04	79	20	73.5	68	7
	6	6.02	44.9	34.2	0.98	76	16	72.0	68	9
	7	7.34	55.4	37.8	1.38	68	19	68.0	63	5
	8	8.20	53.3	36.7	1.70	69	21	68.5	63	3
	9									
	10	9.82	58.0	41.8	1.62	72	16	70.0	66	5
	11									
	12	12.40	65.0	55.0	1.60	85	13	76.5	75	1

TABLE 5 *Continued*

Density Class	D. B. H. Class	Average D. B. H:	Total Height	Fogelberg's Height	Average D. B. T.	Average Form Point	Percent of Bark	Bark Form Class	Bark Form Class	Nº of Samples
Square Feet of Basal Area	Inches	Inches	Feet	Feet	Inches		%			Trees
121 - 160	4	4.07	37.7	30.7	0.73	81	18	74.5	70	3
	5	4.95	40.8	34.8	1.08	85	22	76.5	70	6
	6	5.80	45.0	37.0	1.17	82	20	75.0	70	3
	7	7.30	53.3	40.0	1.20	75	16	71.5	68	3
	8	7.83	53.7	40.8	1.52	76	19	72.0	67	6
	9	9.23	54.0	41.7	1.50	77	16	72.5	69	3
	10	10.10	55.0	42.0	1.70	76	17	72.0	68	1

TABLE 6

SAMPLE TREE DATA USED FOR GROWTH PREDICTION

Density Class	Class D. B. H.	Average D. B. H.	Average D. B. T.	Average D. B. H. Inside Bark	Average Diameter Growth	Average D. B. H. Inside Bark 5 Years Ago	Average D. B. H. Outside Bark 5 Years Ago	Average D. B. H. Growth Outside Bark During Past	Number of Years Grow 1" in D. B. H. Outside Bark	Number of Samples	Weighted Number of Years to Grow 1" in Diameter O. B.
Square Feet of Basal Area	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Years	Trees	Years
21 - 60	4	4.10	0.70	3.40	0.40	3.00	3.55	0.47	10.64	1	10.64
	5	4.90	1.10	3.80	0.60	3.20	3.79	0.71	7.04	1	7.04
	6	6.43	1.13	5.30	0.74	4.56	5.40	0.88	5.68	3	17.04
	7	7.20	1.23	5.97	0.66	5.31	6.29	0.78	6.41	3	19.23
	8	8.07	1.31	6.76	0.82	5.94	7.03	0.97	5.15	7	36.05
	9	8.93	1.30	7.63	0.84	6.79	8.04	0.99	5.05	9	45.45
	10	10.00	1.63	8.37	0.86	7.51	8.89	1.02	4.90	7	34.30
	11	11.37	1.50	9.87	1.14	8.73	10.34	1.35	3.70	3	11.10
	12										
	13	13.3	1.60	11.70	0.60	11.10	13.14	0.71	7.04	1	7.04
		304.16		256.90						35	187.89

Ratio $\dfrac{OB}{IB} = \dfrac{304.16}{256.90} = 1.1840$ $\dfrac{187.89}{35} = 5.37$ years

TABLE 6 Continued

Density Class	D.B.H. Class	Average D.B.H.	Average D.B:T:	D.B:H: Inside Bark	Diameter Growth	Average D.B:H: Inside Bark 5 Years Ago	Average D:B:H: Outside Bark 5 Years Ago	Average D:B:H: Growth Outside Bark During Past	Number of Years Grow 1" in D.B.H. Outside Bark	Number of Samples	Weighted Number of Years to Grow 1" in Diameter O.B.
Square Feet of Basal Area	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Years	Trees	Years
61 - 80	4	4.35	1.35	3.00	0.60	2.40	2.85	0.71	7.04	2	10.48
	5	5.13	1.08	4.05	0.50	3.55	4.22	0.59	8.47	6	50.82
	6	5.89	1.07	4.82	0.54	4.28	5.08	0.64	7.81	7	54.67
	7	6.97	1.19	5.78	0.64	5.14	6.10	0.76	6.58	15	98.70
	8	8.06	1.25	6.81	0.68	6.13	7.28	0.81	6.17	10	61.70
	9	8.99	1.21	7.78	0.62	7.16	8.50	0.74	6.76	9	60.84
	10	10.04	1.40	8.64	0.70	7.94	9.43	0.83	6.02	8	48.16
	11	11.14	1.64	9.50	0.82	8.68	10.31	0.97	5.15	8	41.20
	12	12.13	1.73	10.40	0.74	9.66	11.47	0.88	5.68	3	17.04
	13	12.75	1.90	10.85	0.50	10.35	12.29	0.59	8.47	2	16.94
		578.10		486.88						70	464.15

Ratio $\dfrac{OB}{IB} = \dfrac{578.10}{486.88} = 1.1874$ $\dfrac{464.15}{70} = 6.63$ years

Table 6 Continued

Density Class	D. B. H. Class	Average D. B. H.	Average D. B. T.	Average D. B: H: Inside Bark	Average Diameter Growth	Average D. B: H: Inside Bark 5 Years Ago	Average D. B: H: Outside Bark 5 Years Ago	Average D: B: H: Growth Outside Bark During Past	Number of Years Grow 1" in D. B. H. Outside Bark	Number of Samples	Weighted Number of Years to Grow 1" in Diameter O. B.
Square Feet of Basal Area	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Years	Trees	Years
81 - 100	4	4.15	0.70	3.45	0.44	3.01	3.52	0.52	9.62	4	38.48
	5	5.12	0.95	4.17	0.54	3.63	4.25	0.63	7.94	4	31.76
	6	6.03	1.17	4.86	0.54	4.32	5.06	0.63	7.94	3	23.82
	7	7.11	1.21	5.90	0.68	5.22	6.11	0.80	6.25	9	56.25
	8	8.06	1.27	6.79	0.76	6.03	7.06	0.89	5.62	11	61.82
	9	9.11	1.30	7.81	0.76	7.05	8.26	0.89	5.62	14	78.68
	10	10.05	1.32	8.73	0.74	7.99	9.36	0.87	5.75	16	92.00
	11	10.99	1.41	9.58	0.68	8.90	10.42	0.80	6.25	7	43.75
	12	11.93	1.47	10.46	0.60	9.86	11.55	0.70	7.14	6	42.84
	13	13.10	1.97	11.13	0.70	10.43	12.21	0.82	6.10	4	24.40
	14	14.40	2.50	11.90	0.80	11.10	13.00	0.94	5.32	1	5.32
	15										
	16										
	17	16.90	3.00	13.90	0.80	13.10	15.34	0.94	5.32	1	5.32
		728.37		622.01						80	504.44

Ratio $\dfrac{OB}{IB} = \dfrac{728.37}{622.01} = 1.1710$

$\dfrac{504.44}{80} = $ years

TABLE 6 Continued

Density Class	D. B. H. Class	Average D. B. H.	Average D. B: T:	Average D. B: H: Inside Bark	Average Diameter Growth	Average D. B: H: Inside Bark 5 Years Ago	Average D: B: H: Outside Bark 5 Years Ago	Average D: B: H: Growth Outside Bark During Past	Number of Years Grow 1" in D. B. H. Outside Bark	Number of Samples	Weighted Number of Years to Grow 1" in Diameter O. B.
Square Feet of Basal Area	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Years	Trees	Years
101 - 120	4	4.15	0.75	3.40	0.42	2.98	3.63	0.51	9.80	10	98.00
	5	5.14	1.04	4.10	0.42	3.68	4.48	0.51	9.80	7	68.60
	6	6.02	0.98	5.04	0.46	4.58	5.57	0.56	8.93	9	80.37
	7	7.34	1.38	5.96	0.72	5.24	6.37	0.88	5.68	5	28.40
	8	8.20	1.70	6.50	0.74	5.76	7.01	0.90	5.56	3	16.68
	9										
	10	9.82	1.62	8.20	0.72						
	11											
	12	12.40	1.60	10.80	1.00	9.80	11.92	1.22	4.10	1	4.10
		254.46		209.16						40	324.55

Ratio $\dfrac{OB}{IB} = \dfrac{254.46}{209.16} = 1.2136$ $\qquad \dfrac{324.55}{40} = 8.11$ years

TABLE 6 Continued

Density Class	D. B. H. Class	Average D. B. H.	Average D. B: T.	Average D. B: H: Inside Bark	Average Diameter Growth	Average D. B: H: Inside Bark 5 Years Ago	Average D. B: H: Outside Bark 5 Years Ago	Average D: B: H: Growth Outside Bark During Past	Number of Years Grow 1" in D. B. H. Outside Bark	Number of Samples	Weighted Number of Years to Grow 1" in Diameter O. B.
Square Feet of Basal Area	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Inches	Years	Trees	Years
121 - 160	4	4.07	0.73	3.34	0.40	2.94	3.62	0.49	10.20	3	30.60
	5	4.85	1.08	3.87	0.50	3.37	4.15	0.62	8.06	6	48.36
	6	5.80	1.17	4.63	0.46	4.17	5.13	0.57	8.77	3	26.31
	7	7.30	1.20	6.10	0.60	5.50	6.77	0.74	6.76	3	20.28
	8	7.83	1.52	6.31	0.66	5.65	6.95	0.81	6.17	6	37.02
	9	9.23	1.50	7.73	0.80	6.93	8.53	0.98	5.10	3	15.30
	10	10.10	1.70	8.40	0.60	7.80	9.60	0.74	6.76	1	6.76
		165.98		134.88						25	184.63

Ratio $\dfrac{OB}{IB} = \dfrac{165.98}{134.88} = 1.2306$ $\qquad$ $\dfrac{184.63}{25} = 7.39$ years

Volume Tables for Standard Method

The following volume formula, developed by Swinford (13), from volume curves derived by Fogelberg (8), was used to determine the volume of the sample trees.

$$\text{Volume in cords per tree} = 0.003881 \left(\frac{\text{F.D.}^2\text{H.}}{100} \right) - 0.004064$$

Where:

F = Outside bark form class of the tree expressed as a percentage

D^2 = Diameter breast high in inches, squared

H = Total height in feet

This formula was applied to every diameter class of every density class of the sample tree data. The volumes calculated were plotted and the reading from the corresponding graph was recorded in volume tables.

For computation of growth in cords per acre Table 5 in this Appendix provides the corresponding volume tables for every density class. The usual procedure is to read from the average d.b.h. in inches the corresponding volume and then multiplying this volume by the average number of trees per acre.

Volume Tables for V.P.R. Method

In this case, the volume in cords is expressed as volume per square foot of basal area. The appropriate volume factors are found by dividing the volume in cords of each diameter class, as expressed in the regular volume tables, by their own basal area.

When working completely with the V.P.R. method, as in the second situation, the volume in cords per acre is found as follows:

1. When making the count, tally the count trees in diameter classes.

2. Multiply the count trees in each diameter class by their corresponding volume factor.

3. The sum of the products times the B.A.F. employed divided by the total number of plots, will give the volume in cords per acre.

For the conversion of the basal area growth to volume growth in cords per acre, the procedure is somewhat more complicated. The steps to follow are:

1. Tally the count trees by diameter classes.

2. Derive the average stand table for the stand in question by the following procedure:

a) Find the basal area in square feet per acre represented by each diameter class:

$$\frac{Trees\ in\ a\ given\ diameter\ class \times B.A.F.}{Total\ number\ of\ plots}$$

b. This basal area, divided by the basal area corresponding to one tree of that diameter class will give the number of trees in that diameter class.

Continue the same procedure through the entire range of the data.

3. Once the stand table has been derived, find the average d.b.h. of the stand by computing a weighted average of the stand by computing a weighted average of the diameter classes represented in the stand table.

4. This average diameter serves as the basis for entering the table of volume factors to obtain the appropriate volume per square feet of basal area. This, multiplied by the basal area growth per acre will give volume growth in cords per acre. The volume factors for the data used in this study can be found in Table 8.

TABLE 7

VOLUME TABLES FOR STANDARD METHOD
CORDS PER TREE

D.B.H. Class (inches)	Density Classes (square feet of basal area/acre)				
	21 — 60	61 — 80	81 — 100	101 — 120	121 — 160
4	0.012	0.008	0.007	0.012	0.011
5	0.024	0.021	0.018	0.023	0.024
6	0.038	0.037	0.035	0.037	0.041
7	0.055	0.057	0.058	0.056	0.062
8	0.076	0.080	0.086	0.081	0.086
9	0.100	0.107	0.118	0.112	0.114
10	0.129	0.139	0.157	0.149	0.144
11	0.162	0.176	0.202	0.195	0.178
12	0.200	0.217	0.250	0.248	0.217
13	0.243	0.265	0.301	0.311	0.259
14	0.292	0.317	0.358	0.383	0.306
15	0.347	0.377	0.415	0.466	0.357
16	0.401	0.444	0.479	0.560	0.415
17	0.478			0.663	0.476

SOURCE: Data from Austin Cary Memorial Forest, Gainesville, Florida.
SPECIES: Slash Pine (*Pinus elliottii,* var. *elliottii* Engelm.)
DATE: November, 1963.
No. OF SAMPLE TREES: 250.

TABLE 8

VOLUME TABLES FOR V.P.R. METHOD
(VOLUME PER SQUARE FOOT OF BASAL AREA)

D.B.H. Class (inches)	Density Classes (square feet of basal area/acre)				
	21 − 60	61 − 80	81 − 100	101 − 120	121 − 160
4	0.141	0.090	0.077	0.139	0.128
5	0.176	0.153	0.132	0.168	0.176
6	0.196	0.190	0.178	0.190	0.209
7	0.207	0.212	0.216	0.211	0.233
8	0.216	0.228	0.245	0.232	0.247
9	0.226	0.243	0.268	0.253	0.258
10	0.236	0.255	0.288	0.274	0.265
11	0.246	0.266	0.306	0.295	0.270
12	0.255	0.277	0.378	0.316	0.276
13	0.264	0.287	0.324	0.337	0.281
14	0.273	0.297	0.335	0.358	0.286
15	0.283	0.307	0.338	0.380	0.291
16	0.293	0.318	0.343	0.401	0.297
17	0.303			0.421	0.302

SOURCE: Austin Cary Memorial Forest.
SPECIES: Slash Pine (*Pinus elliottii,* var. *elliottii* Engelm.)
DATE: November, 1963.
NUMBER OF SAMPLE TREES: 250.

TABLE 9

GROWTH' IN BASAL AREA PER ACRE AS PREDICTED BY ABREVIATED AND POINT CENTER EXTENSION METHODS

Plot Number	Abreviated Method	Point Center Extension Method
1	16.75	20
2	13.96	20
3	10.33	20
4	13.42	40
5	15.90	20
6	19.62	20
7	22.00	30
8	14.03	10
9	24.22	30
10	18.48	20
11	19.47	10
12	26.93	20
13	19.87	10
14	16.96	30
15	13.64	20
16	18.85	20
17	30.77	40
18	15.04	40
19	19.15	10
20	21.05	20
21	14.05	20
22	19.48	30
23	12.64	10
24	17.93	20
25	23.44	30
26	19.54	30

TABLE 9 *Continued*

Plot Number	Abreviated Method	Point Center Extension Method
27	19.95	30
28	45.92	40
29	19.13	20
30	14.91	10
31	15.67	10
32	11.69	10
33	9.98	20
34	19.31	20
35	15.32	10
36	13.85	10
37	20.15	40
38	26.19	20
39	20.88	10
40	13.86	10
41	16.59	10
42	21.96	40
43	23.66	30
44	29.48	20
45	33.59	30
46	33.72	20
47	35.39	30
48	9.82	30
49	9.83	20
50	7.24	20

TABLE 10

GROWTH IN BASAL AREA PER ACRE PREDICTED VS. ACTUAL RESULTS

$$Y = 8.37572 - 0.12982X_1 + 0.04317X_2$$

(FIRST SITUATION)

Plot Number	Actual	Predicted	Deviation
1	16.75000	15.10404	1.64596
2	13.96000	14.58600	— 0.62600
3	10.33000	10.95972	— 0.62772
4	13.42000	12.68032	0.73968
5	15.9000	13.37724	2.52276
6	19.62000	17.69424	1.92576
7	22.00000	21.57644	0.42356
8	14.03000	14.84812	— 0.81812
9	24.22000	23.99396	0.22604
10	18.48000	19.76640	— 1.28640
11	19.47000	18.99244	0.47756
12	27.93000	25.29216	2.63784
13	19.87000	18.81976	1.05024
14	16.96000	14.15120	2.80880
15	13.64000	15.27662	— 1.63672
16	18.85000	19.07568	— 0.22568
17	30.77000	34.61068	— 3.84068
18	15.04000	16.57492	— 1.53492
19	19.15000	16.40224	2.74776
20	21.05000	19.59372	1.45628
21	14.05000	13.72260	0.32740
22	19.48000	20.36768	— 0.88768
23	12.64000	15.36616	— 2.72616
24	17.93000	21.32052	— 3.39052

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Table 10 *Continued*

Plot Number	Actual	Predicted	Deviation
25	23.44000	18.29557	5.14448
26	14.54000	13.97852	0.561480
27	19.95000	21.05840	— 1.10840
28	45.92000	45.83488	0.08512
29	19.13000	18.21228	0.81772
30	14.91000	14.58910	0.32090
31	15.67000	15.36616	0.30384
32	11.69000	13.81204	— 2.12204
33	9.98000	9.57828	0.40172
34	19.31000	17.69424	1.61576
35	15.32000	15.02080	0.29920
36	13.82000	13.98472	— 0.13472
37	20.15000	21.65968	— 1.50968
38	26.19000	23.04732	3.14268
39	20.88000	19.85584	1.02416
40	13.86000	17.43832	— 3.57832
41	16.59000	18.64708	— 2.04708
42	21.96000	25.80400	— 3.84400
43	23.66000	26.06612	— 2.40612
44	29.48000	29.26380	0.21620
45	33.59000	32.28260	1.37040
46	33.72000	33.06276	0.65724
47	35.39000	32.10992	3.28008
48	9.82000	9.14348	0.67652
49	9.83000	10.95972	— 1.12972
50	7.24000	10.61436	— 3.37436

Y = Growth in basal area per acre.
X_1 = Growth in basal area per acre as computed by the point center extension method.
X_2 = Number of trees per acre as computed by standard inventory procedures.

TABLE 11

GROWTH IN BASAL AREA PER ACRE PREDICTED VS. ACTUAL RESULTS

$$Y = 3.36972 + 0.15148X_1 + 0.14918X_5$$

Plot Number	Actual	Predicted	Deviation
1	16.75000	16.84206	— 0.09206
2	31.96000	15.35023	— 1.39023
3	10.33000	13.85840	— 3.52840
4	13.42000	18.37978	— 4.95978
5	15.9000	18.33389	— 2.43389
6	19.62000	19.82572	— 0.20572
7	22.00000	18.35683	3.64317
8	14.03000	15.32729	— 1.29729
9	24.22000	18.35683	5.86316
10	18.48000	24.30120	— 5.82120
11	19.47000	18.31094	1.15906
12	27.93000	19.82572	8.10428
13	19.87000	19.80277	0.06723
14	16.96000	19.84866	— 2.88866
15	13.64000	15.35023	— 1.71023
16	18.85000	16.84206	2.00794
17	30.77000	24.34709	6.42291
18	15.04000	12.34363	2.69637
19	19.15000	12.34363	6.80637
20	21.05000	18.33389	2.71611
21	14.05000	13.85840	0.19160
22	19.48000	21.34049	— 1.86094
23	12.64000	16.81911	— 4.17911
24	17.93000	22.80937	— 4.87937

TABLE 11 *Continued*

Plot Number	Actual	Predicted	Deviation
25	23.44000	22.83232	0.60768
26	14.54000	21.34049	— 6.80049
27	19.95000	25.81597	— 5.86597
28	45.92000	31.80623	14.11377
29	19.13000	18.33389	0.79611
30	14.91000	15.32729	— 0.41729
31	15.67000	16.81911	— 1.14911
32	11.69000	16.81911	— 5.12911
33	9.98000	16.84206	— 6.86206
34	19.31000	24.30120	— 4.99120
35	15.32000	24.27825	— 8.95825
36	13.85000	12.34363	1.50637
37	20.15000	31.80623	—11.65623
38	26.19000	15.35023	10.83977
39	20.88000	18.31094	2.56906
40	13.86000	10.85180	3.00820
41	16.59000	12.34363	4.24637
42	21.96000	25.83892	— 3.87892
43	23.66000	24.32415	— 0.66415
44	29.48000	27.28486	2.19514
45	33.59000	28.79963	4.79037
46	33.72000	25.79303	7.92697
47	35.39000	27.30780	8.08220
48	9.82000	13.88135	— 4.06135
49	9.83000	10.87475	— 1.04475
50	7.24000	10.87475	— 3.63475

Y = Growth in basal area per acre.
X_1 = Growth as predicted by the point center extension method.
X_2 = Basal area in square feet per acre as computed by V.P.R. method.

TABLE 12

SUMMARY TABLE

NUMBER OF TREES PER ACRE, BASAL AREA PER ACRE STANDARD METHOD; AVERAGE D.B.H., BASAL AREA PER ACRE V.P.R. METHOD

Plot	Trees Number of Trees	Square Feet Basal Area/Acre	Inches Average	Square Feet Basal Area/Acre
Plot Number	Per Acre	Standard Method	D. B. H:	V: P: R: Method
1	216	83.02	8.4	70
2	204	67.84	7.8	60
3	120	44.66	8.3	50
4	220	60.79	7.1	60
5	176	83.11	9.3	80
6	276	89.51	7.7	90
7	396	75.14	5.9	70
8	180	69.90	8.4	70
9	452	79.90	5.7	70
10	324	114.52	8.1	120
11	276	90.34	7.7	90
12	452	94.34	6.2	90
13	272	100.21	8.2	100
14	224	81.59	8.2	80
15	220	68.45	7.6	60
16	308	77.46	6.8	70
17	728	112.81	5.3	100
18	220	63.36	7.3	50
19	216	54.83	6.8	50
20	320	83.44	6.9	80
21	184	74.97	8.6	50
22	368	108.45	7.4	90
23	192	68.63	8.1	80

Plot Number	Per Acre Number of Trees	Standard Method Basal Area/Acre	D. B. H: Average	V: P: R: Method Basal Area/Acre
Plot	Trees	Square Feet	Inches	Square Feet
24	360	108.14	7.4	110
25	320	96.96	7.5	100
26	220	74.21	7.9	90
27	384	106.22	7.1	120
28	988	151.76	5.3	150
29	288	86.76	7.4	80
30	174	82.63	9.3	70
31	192	89.71	9.2	80
32	156	69.90	9.1	80
33	88	51.07	10.3	70
34	276	94.04	7.9	120
35	184	84.57	9.2	130
36	160	48.32	7.4	50
37	428	101.15	6.6	150
38	400	88.13	6.4	60
39	296	89.41	7.4	90
40	240	61.86	6.9	40
41	268	64.02	6.6	50
42	524	109.31	6.2	110
43	500	111.13	6.4	110
44	544	136.38	6.8	140
45	644	149.88	6.5	140
46	632	134.47	6.3	130
47	640	134.70	6.2	130
48	108	42.54	8.5	40
49	120	41.26	7.9	30
50	112	36.05	7.7	30